

STAT 2060
Summer 2016
Midterm
9-Jun-16
Time Limit: 60 Minutes

Name (Print): Total 50

Student ID Solution

This exam contains 4 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

- 8 1. A common injury for downhill ski racers is to the anterior cruciate ligament (ACL) of the knee. The number of ACL operations, X , required by a ski racer during their career is a random variable with probability distribution shown below.

Table 1

x	0	1	2
$p(x)$	0.1	0.5	0.4

- (a) (4 points) Find expected value and variance of X .

$$EX = 0(0.1) + 1(0.5) + 2(0.4) = 1.3$$

$$EX^2 = 0^2(0.1) + 1^2(0.5) + 2^2(0.4) = 2.1$$

$$\text{Var}(X) = EX^2 - (EX)^2 = 2.1 - 1.3^2 = 0.41$$

- (b) (4 points) Find expected value of $4X - 1$.

$$E(4X - 1) = 4EX - 1 = 4(1.3) - 1 = 4.2$$

$$\text{Var}(4X - 1) = 4^2 \text{Var}(X) = 4^2(0.41) = 6.56$$

2. (4 points) A computer repair technician mistakenly puts five faulty motherboards into a box of 20 good motherboards. The bad parts are indistinguishable from the good parts. If two parts are selected at random (without replacement). What is the probability that one is good and the other is bad.

$$\frac{\binom{5}{1} \binom{20}{1}}{\binom{25}{2}} = \frac{5 \times 20}{\frac{25 \times 24}{2 \times 1}} = \frac{100}{1300} = \frac{1}{13}$$

3. The windshield wiper fluid reservoir for many automobiles has a 4 liter capacity. A randomly chosen car has wiper fluid, X , with the density function $f(x) = \frac{1}{16} + \frac{3x}{32}$ for $0 \leq x \leq 4$.

- (a) (4 points) What is the cumulative distribution function for X ?

$$\begin{aligned} F(x) = P(X \leq x) &= \int_{-\infty}^x f(x) dx & \Rightarrow F(x) &= \begin{cases} 0 & x < 0 \\ \frac{1}{16}x + \frac{3x^2}{64} & 0 \leq x \leq 4 \\ 1 & x > 4 \end{cases} \\ &= \int_0^x \left(\frac{1}{16} + \frac{3x}{32} \right) dx \\ &= \left(\frac{1}{16}x + \frac{3}{32} \left(\frac{1}{2}x^2 \right) \right) \Big|_0^x \\ &= \frac{1}{16}x + \frac{3x^2}{64} \end{aligned}$$

- (b) (3 points) What is the probability a car has more than 3 liters of fluid in its reservoir?

$$\begin{aligned} P(X > 3) &= 1 - P(X \leq 3) \\ &= 1 - F(3) \\ &= 1 - \left(\frac{3}{16} + \frac{27}{64} \right) \\ &= \frac{25}{64} = 0.3906 \end{aligned}$$

- (c) (4 points) What is the median amount of wiper fluid in a car?

$$\begin{aligned} F(x_0) &= 0.5 & \Rightarrow 0 \leq x_0 \leq 4 \\ \frac{1}{16}x_0 + \frac{3}{64}x_0^2 &= \frac{1}{2} & \text{Median value is } x_0 = \frac{8}{3} \\ 4x_0 + 3x_0^2 &= 32 = 0 \\ (3x_0 - 8)(x_0 + 4) &= 0 \\ x_0 &= \frac{8}{3} \text{ or } x_0 = -4 \end{aligned}$$

4. Accidents on highway 103 occur according to a Poisson process at the rate of 1.5 per week.

11 (a) (3 points) What is the probability that there is exactly one accident in a week?

$$X \sim \text{Pois}(1.5)$$

$$\begin{aligned} P(X=1) &= \frac{e^{-\lambda} \lambda^x}{x!} \\ &= \frac{e^{-1.5} 1.5^1}{1!} = 0.3347 \end{aligned}$$

(b) (4 points) What is the probability that there is at least one accident in a week?

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X=0) \\ &= 1 - \frac{e^{-1.5} 1.5^0}{0!} \\ &= 1 - e^{-1.5} = 0.7769 \end{aligned}$$

(c) (4 points) What is the probability that there are two accidents in two week?

$$X \sim \text{Pois}(3)$$

$$P(X=2) = \frac{e^{-3} 3^2}{2!} = 0.224$$

8 5. Suppose that 25% of all students who want a new copy of textbook for a particular course, whereas 75% want a used copy. Consider randomly selecting 10 purchasers.

(a) (4 points) What is the probability that the number who want new copies is at least 2?

$$X \sim \text{Binomial}(10, 0.25)$$

iz 1P
found $P(X=2)$
using correct
formula

$$\begin{aligned} P(X \geq 2) &= 1 - P(X < 2) \\ &= 1 - P(X=0) - P(X=1) \\ &= 1 - \binom{10}{0} 0.25^0 0.75^{10} - \binom{10}{1} 0.25^1 0.75^9 \\ &= 1 - 0.0563 - 0.188 \\ &= 0.756 \end{aligned}$$

- (b) (4 points) What is the expected value and standard deviation of the number who want a new copy?

$$EX = np = 10 \times 0.25 = 2.5$$

$$SD(X) = \sqrt{np(1-p)} = \sqrt{10 \times (0.25)(0.75)} = 1.369$$

no sart $\left(\frac{1}{2}\right)$
 wrong p $\left(\frac{1}{2}\right)$

6. At a certain gas station, 40% of customers use regular gas, 35% use plus gas, and 25% use premium gas. Of those customers using regular gas, only 30% fill their tanks. Of those customers using plus gas, only 60% fill their tanks, whereas of those using premium, 50% fill their tanks.

- (a) (4 points) What is the probability that the next customer will request plus gas and fill the tank?

$A_1 = \{ \text{customers use regular gas} \}$

$$P(A_1) = 0.4$$

$$P(B|A_1) = 0.3$$

$A_2 = \{ \text{customers use plus gas} \}$

$$P(A_2) = 0.35$$

$$P(B|A_2) = 0.6$$

$A_3 = \{ \text{customers use premium gas} \}$

$$P(A_3) = 0.25$$

$$P(B|A_3) = 0.5$$

$B = \{ \text{customers requests fill tank} \}$

$$\Rightarrow P(B \cap A_2) = P(B|A_2)P(A_2)$$

$$= 0.6 \times 0.35$$

$$= 0.21 \quad \uparrow \text{use } P(B|A_2)$$

- (b) (4 points) What is the probability that the next customer fills the tank? (Indication:

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3))$$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3)$$

$$= 0.3(0.4) + 0.6(0.35) + 0.5(0.25)$$

$$= 0.455$$