Name: SOLUTIONS Student Number: ________________________

STATISTICS 2060/MATH 2060/ECON 2260 Midterm Test
Thursday, Feb. 26, 2015

Please answer the questions in the space provided. Justify your answers. A calculator, and one sheet of formulae/notes, letter size, are permitted.

1. The probability of rain during a winter nor'easter storm in Halifax is .60, and the probability of a 'flash freeze', or sudden steep drop in temperature, is .20. The probability of a flash freeze if there has been rain is .40.

(3) (a) Find the probability that the next nor'easter storm has rain and a flash freeze.

$P(R \cap F) = P(F|R)P(R) = .4(.6) = .24$

(3) (b) Find the probability that the next nor'easter storm has rain or a flash freeze.

$P(R \cup F) = P(R) + P(F) - P(R \cap F)$

$= .6 + .2 - .24 = .56$

(2) (c) Are the events "rain" and "flash freeze" independent? Justify your answer.

$P(F|R) = .4 \neq P(F) = .2$ so not independent

(2) (d) Are the events "rain" and "flash freeze" mutually exclusive? Justify your answer.

No. $P(R \cap F) = .24 \neq 0$ so $R \cap F \neq \phi$

2. It is known that 5% of the pills made by a certain machine are chipped. The pills are packaged 12 to a box. Cumulative probabilities for the binomial with $n = 12$ and $p = .05$ are as follows:

x	0	1	2	3	4	5	...	12
$P(X \leq x)$	.5404	.8816	.9804	.9977	.9998	1.0000	...	1.0000

(3) What is the probability that the number X which are chipped is between 1 and 3, (inclusive), ie that $P(1 \leq X \leq 3)$?

$P(X \leq 3) - P(X = 0) = .9977 - .5404 = .4573$

3. On an icy winter day, the number of people, X, arriving at the QEII hospital in Halifax with broken bones after falling has the following probability distribution:

x	0	1	2
$p(x)$	.3	.5	.2

(3) (a) What is the mean number of people arriving with broken bones?

$E(X) = 0(.3) + 1(.5) + 2(.2) = .5 + .4 = .9$

(3) (b) What is the standard deviation of the number of people arriving with broken bones?

$E(X^2) = 0(.3) + 1(.5) + 4(.2) = .5 + .8 = 1.3$

$V(X) = 1.3 - .9^2 = .49$

$\sigma_X = .7$

(2) (c) Suppose the cost of treating each patient is $1500. What are the mean and standard deviation of the cost per day of treating patients with broken bones from falling?

$\mu_C = 1500 \times .9 = \$1350.00$

$\sigma = 1500(.7) = \$1050.00$

4. The number of songs downloaded from a particular music site follows a Poisson process with mean 2 songs per minute.

(3) (a) What is the probability that 3 songs are downloaded in one minute?

$P(X=3) = \frac{e^{-2} 2^3}{3!} = \frac{.1353(8)}{6} = .1804$

(3) (b) What is the probability that exactly one song is downloaded in a 4 minute interval?

$P(Y=1) = \frac{e^{-8} 8^1}{1} = .000335(8) = .00268$

5. Three guests come to dinner, leaving their hats by the front door. When the time comes to leave, the host can't remember which hat belongs to each person, and randomly gives one to each.

(3) (a) List the sample space for this experiment. Each outcome should be given as a triple of three letters, with the position representing the person and the letter representing the hat. For example *bca* inticates that Mr. A gets Mr. B's hat, Mr. B gets Mr. C's hat. and Mr. C. gets Mr. A.'s hat.

$S = \{abc, acb, bac, bca, cab, cba\}$

(4) (b) Define the random variable, X, to be the number of hats given to the correct owner. List the possible values for X, and give their probabilities.

		prob
$X=0$ when we get	bca cab	2/6
$X=1$	acb bac cba	3/6
$X=3$	abc	1/6

6. The time in days, X, required to repair computers at a particular shop has exponential probability density

$$f(x) = \frac{2}{3}e^{-2x/3}$$

for $x > 0$

(2) (a) What is the probability that a repair takes more than 3 days?

$e^{-2(3)/3} = e^{-2} = .1353$

(2) (b) Given that a repair has taken more than 2 days, what is the probability that it takes more than 5 days?

same as above $e^{-2} = .1353$

due to memoryless property of the exponential.

(3) (c) What is the time by which 75% of the repairs have been made?

$$F(x) = 1 - e^{-2x/3}$$

want $F(\eta) = 1 - e^{-2\eta/3} = .75$

or $1 - .75 = e^{-2\eta/3}$

$$\ln(.25) = -2\eta/3$$

$$\eta = -\frac{3}{2}\ln(1/4) = \frac{3}{2}\ln(4) = \frac{3}{2}(1.3863) = 2.079$$

(3) 7. A gardener plants 6 bulbs selected at random from a box of 8 tulip and 4 daffodil bulbs. What is the probability he planted exactly 2 daffodil bulbs?

$$\frac{\binom{4}{2}\binom{8}{4}}{\binom{12}{6}} = \frac{\left(4!/2!2!\right)\left(8!/4!4!\right)}{12!/6!6!} = \frac{6 \times 8\times7\times6\times5/4\times3\times2}{12\times11\times10\times9\times8\times7/6\times5\times4\times3\times2}$$

$$= \frac{420}{132\times7} = \frac{60}{132} = \frac{10}{22} = \frac{5}{11} = .45$$

8. The amount of snowfall in February in Halifax is normally distributed with mean 41 cm and standard deviation 15 cm.

(3) (a) What is the probability of getting more than 60 cm in a February in Halifax?

$$P(X > 60) = P\left(Z > \frac{60-41}{15}\right) = P\left(Z > \frac{19}{15}\right)$$

$$= P(Z > 1.27) = P(Z < -1.27) = .1020$$

(3) (b) What is the 10th percentile of February snow in Halifax?

$$P(X < \eta) = .10$$

$$P\left(Z < \frac{\eta - 41}{15}\right) = .10$$

so $\dfrac{\eta - 41}{15} = -1.28$

or $\eta = 41 - 1.28(15) = 41 - 19.2 = 21.8$

(4) 9. Given the probability density $f(x) = 4x^{\overset{3}{\cancel{2}}}$, $0 < x < 1$, find $P(1/4 < X < 3/4)$.

$$F(x) = \int_0^x 4y^3\,dy = \frac{4y^4}{4}\Big|_0^x = x^4 \qquad 0 < x < 1$$

so $P\left(\frac{1}{4} < X < \frac{3}{4}\right) = F\left(\frac{3}{4}\right) - F\left(\frac{1}{4}\right) = \left(\frac{3}{4}\right)^4 - \left(\frac{1}{4}\right)^4$

$$= \frac{81 - 1}{256} = \frac{80}{256} = \frac{10}{32} = \frac{5}{16}$$

$$= .3125$$

10. A box contains three coins, two of them fair and the other has two heads. A coin is selected at random and tossed twice.

(3) (a) What is the probability of getting heads on both tosses?

$$P(T) = P(C_1)P(T|C_1) + P(C_2)P(T|C_2) + P(C_3)P(T|C_3)$$
$$= \frac{1}{3}\,\frac{1}{4} + \frac{1}{3}\,\frac{1}{4} + \frac{1}{3}(1)$$
$$= \frac{1}{3}\left(\frac{3}{2}\right) = \frac{1}{2}$$

(3) (b) If you get two heads, what is the probability that the coin is the two-headed coin?

$$P(C_3|T) = \frac{P(T|C_3)P(C_3)}{P(T)} = \frac{1\left(\frac{1}{3}\right)}{1/2} = \frac{2}{3}$$