One-Sample Hypothesis Tests II(8.3)

1. Small Sample Test of Significance for Population Proportion, p

   (a) **Population Proportion:** The proportion of individuals in a population with a specific attribute, often labeled as S for success.

Population Proportion	Sample Proportion	Sampling Distribution
$p = X_{pop}/N$	$\hat{p} = X/n$	Small n: Binomial Large n: Normal

   - The Null Hypothesis takes the form:

$$H_0 : p = p_0$$

     where p_0 is a hypothesized numerical value for population proportion.
   - The Alternative Hypothesis is chosen from the three possible alternative hypotheses for use with the above null hypothesis:

$$H_A : p \neq p_0 \;\; ; \;\; H_A : p < p_0 \;\; ; or \;\; H_A : p > p_0$$

   - **Conclusion:** Only when sample data strongly suggest that p equals something other than p_0 should the null hypothesis be rejected.

2. **Small Sample Test Statistic:** The test statistic is formulated based on the sampling distribution for X, the count of the number of successes in the sample. The sampling distribution for X is binomial with $p = p_0$ and n equal to the sample size.

   (a) When H_0 is true: $X \sim b(n, p_0)$

   (b) First identify the rejection region(s) boundary, C (or C_1 and C_2).

Type Test	Rejection Region(s)	Solve for C (or C_1 and C_2)
$H_A : p > p_0$	$X \geq C$	$1 - \sum_{x=0}^{C-1} b(x, n, p_0) \leq \alpha$
$H_A : p < p_0$	$X \leq C$	$\sum_{x=0}^{C} b(x, n, p_0) \leq \alpha$
$H_A : p \neq p_0$	$X \geq C_1$ $X \leq C_2$	$1 - \sum_{x=0}^{C_1-1} b(x, n, p_0) \leq \alpha/2$ $\sum_{x=0}^{C_2} b(x, n, p_0) \leq \alpha/2$

   (c) Then compare values of X with C, reject H_0 when the value of X lies within the rejection region.

3. Example: We have fed a diet high in saccharin to a sample of 20 rats and find that 2 of them develop bladder cancer. In our lab, historically 5.0% of all rats tested develop bladder cancer regardless of diet. Is our finding significantly greater than, at $\alpha = .05$, the diet neutral historical proportion?

$$H_0 : p = .05$$

$$H_A : p > .05$$

- Let rv X denote the count of rats in the sample that develop bladder cancer.

- Sample Information: $n = 20$ and $x = 2$

- Point Estimate of p: $\hat{p} = \frac{x}{n} = \frac{2}{20} = 0.10$

- When H_0 is true, $X \sim b(20, .05)$

- Check for the Normal approximation: $n\hat{p}(1 - \hat{p}) \geq 10$, here $20(.05)(.95) = 0.95$. So Normal not a good approximation here.

- Test Statistic: Use binomial X as test statistic, determine C, the boundary of the rejection region. Reject H_0 if X lies in rejection region.

- Here, $H_A : p > p_0$, so the rejection region is $X \geq C$.

- Determine C from $1 - \sum_{x=0}^{C-1} b(x, 20, .05) \leq .05$.

- or $\sum_{x=0}^{C-1} b(x, 20, .05) \geq .95$

- Let $d = C - 1$, then using Table A.1 with $n = 20$
  evaluate $B(d; 20, .05) = \sum_{x=0}^{d} b(x; 20, .05) \geq .95$

- From Table A.1, $d = 3$ is the first value greater than .95 for $\alpha = .05$.
  Choose $d = 3$ as the value we seek.

- Since $d = (C - 1)$, $C = 4$ is the rejection region boundary.

- If sample computed $X \geq C$ then reject H_0. Here, $C = 4$ and $X = 2$ from the problem statement, so X is not greater than or equal to C.

- Conclude: The data do not provide strong enough evidence to reject H_0.